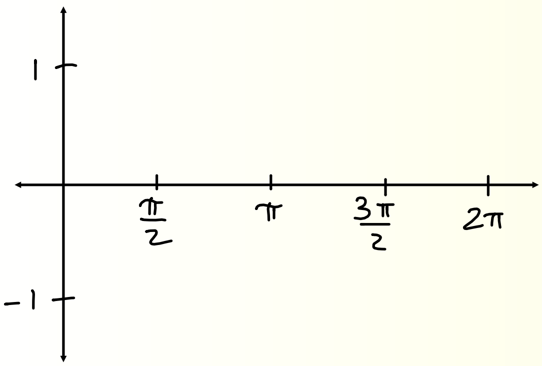
Calculus Warm Up #11-2

1. Sketch one cycle of y=sinx and y=cosx on the same axes.



2. Fill in the table:

x	$\cos x$	$\frac{d}{dx}[\sin x]$
0		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

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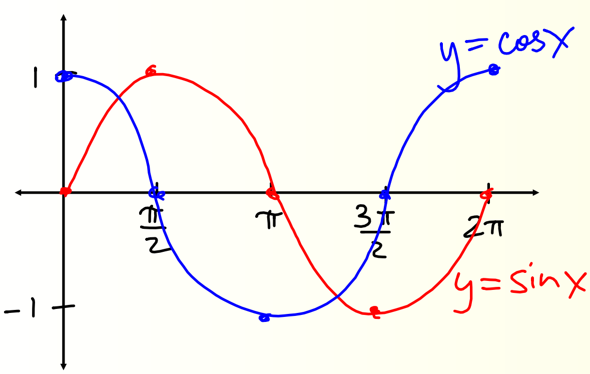


2. Fill in the table:

x	$\cos x$	$\frac{d}{dx}[\sin x]$
0	1	
$\frac{\pi}{2}$	0	
π	-1	
$\frac{3\pi}{2}$	0	
2π	1	

8.3 - Derivatives of trig functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

Find the derivative of $y = \underbrace{2x \cos x} - 2 \sin x$.

$$y' = 2x(-\sin x) + 2\cos x - 2\cos x$$

$$y' = -2x \sin x$$

Find the slope of the graph of $f(x) = 2 \cos x$ at the following points.

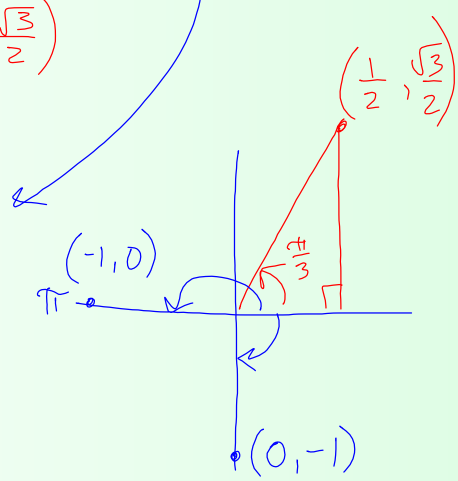
(a) $\left(-\frac{\pi}{2}, 0\right)$ (b) $\left(\frac{\pi}{3}, 1\right)$ (c) $(\pi, -2)$

$f'(x) = -2\sin x$

$$f'\left(-\tfrac{\pi}{2}\right) = -2\sin\left(-\tfrac{\pi}{2}\right)$$
$$= -2(-1)$$
$$= 2$$

$$f'\left(\tfrac{\pi}{3}\right) = -2\sin\left(\tfrac{\pi}{3}\right)$$
$$= -2\left(\tfrac{\sqrt{3}}{2}\right)$$
$$= -\sqrt{3}$$

c) $f'(\pi) = -2\sin\pi$
$$= -2(0)$$
$$= 0$$



Changing the form first:

$$y = \frac{1 - \cot x}{\csc x}$$

$$y = \frac{1}{\csc x} - \frac{\cot x}{\csc x}$$

$$y = \sin x - \frac{\frac{\cos x}{\cancel{\sin x}}}{\frac{1}{\cancel{\sin x}}}$$

$$\frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1}$$

$$y = \sin x - \cos x$$

$$\boxed{y' = \cos x + \sin x}$$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \longrightarrow = \boxed{\sec^2 x}$$

THEOREM 8.4 DERIVATIVES OF TANGENT, COTANGENT, SECANT, AND COSECANT

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

differentiate:

$$y = \tan[e^{3x}]$$

let $u = e^{3x}$

$\frac{du}{dx} = 3e^{3x}$

$$y' = [\sec^2(e^{3x})]\, 3e^{3x}$$
$$= 3e^{3x} \sec^2(e^{3x})$$

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx} \qquad \frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$$

$$\boxed{\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx}} \qquad \frac{d}{dx}[\cot u] = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx} \qquad \frac{d}{dx}[\csc u] = -\csc u \cot u \frac{du}{dx}$$

Be careful of ( ) 's !!

a) $y = \cos 3x^2 = \cos(3x^2)$

$y' = [-\sin(3x^2)](6x)$

$y' = -6x \sin 3x^2$

b) $y = \underbrace{(\cos 3)}_{\text{constant}} x^2$

$y' = (2\cos 3)x$

$= 2x\cos 3$

c) $y = \cos(3x)^2 = \cos(9x^2)$

$y' = (-\sin 9x^2)(18x)$

$y' = -18x \sin 9x^2$

d) $y = \cos^2 3x = (\cos 3x)^2$

Chain Rule!

$y' = (2\cos 3x)(-\sin 3x)\,3$

$y' = -6\cos 3x \sin 3x$

$y' = -3(2\sin 3x \cos 3x)$

$y' = -3\sin 6x$

EXAMPLE 8 Differentiating a composite function

Differentiate

$$f(t) = \sqrt{\sin 4t}.$$

$$f'(t) = \frac{1}{2}(\sin 4t)^{-1/2}(\cos 4t)(4)$$

$$= \frac{2\cos 4t}{\sqrt{\sin 4t}}$$

Finding extrema on a closed interval:

Remember to check the end points!

$f(x) = 2 \sin x - \cos 2x$, on $[0, 2\pi]$

$f'(x) = 2\cos x + 2\sin 2x$

$= 2(\cos x + \sin 2x)$

$= 2[\cos x + 2\sin x\cos x]$

$0 = 2\cos x(1 + 2\sin x)$

$\cos x = 0$ $\qquad$ $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

check outcomes:

$f(0) =$

$f(2\pi) =$

$f(\frac{\pi}{2}) =$

$f(\frac{3\pi}{2}) =$

$f(\frac{7\pi}{6}) =$

$f($

Max $(\frac{\pi}{2}, 3)$ $\qquad$ Min $(\frac{7\pi}{6}, -\frac{3}{2})$

$(\frac{11\pi}{6}, -\frac{3}{2})$

HW: p. 446

1 - 43 odd

(skip # 21 & 37)

AP Rev. WS # 7 due Friday